Dimensional synthesis and optimization of planar linkages for rigid body guidance: A review

Abdul Qaiyum, Aas Mohammad, Hasan Zakir Jafri

Abstract A mechanism.is an arrangement of machine elements that produce a specific motion. In order to achieve this task several mechanisms are proposed depending upon their types and linkages. The most common form is that of planer linkages, these linkages provide complex motion ranging from function generation, path generation and guiding a rigid body at specified path and orientation. The current works aims to study the methods employed on four bar planer mechanisms, their synthesis and optimization. The method is then extended to six bar planer linkages and studies the different optimization methods used for optimizing the link lengths and/or orientation to achive a specific rigid body motion.

Key Points—planer mechanism, four bar mechanism, six bar mechanism, dimensional synthesis, rigid body guidance, optimization

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1 INTRODUCTION

A MECHANISM is defined as an arrangement of machine elements that produce a specified motion. Four-bar mechanisms are a class of simple but practically important mechanisms. Their utilization ranges from simple devices, such as windshield-wiping mechanisms and door-closing mechanisms, to complicated ones, such as rock crushers, sewing machines, round balers, and suspension systems of automobiles. To generate a desired motion and orientation under side conditions, the movement capabilities of four bar linkages are limited. Only two basic types of pin-connected six-link kinematic chains provide mechanisms which give motion of an output crank different from that of a four-bar linkage. These are the Watt and Stephenson kinematic chains. These classifications depend on the placement of the ternary links. In the Watt chain, ternary links are adjacent, in the Stephenson chain, the ternary links are separated by binary links.

The synthesis of a mechanism is the process of combining parametric elements into a mechanism that shows complex behavior. Mechanism synthesis includes function generation, motion generation and path generation. Both graphical and analytical techniques with and without prescribed timing are well studied. The graphical techniques are limited to a finite number of precision points for the obvious reason of constraints of drawing too complex figures

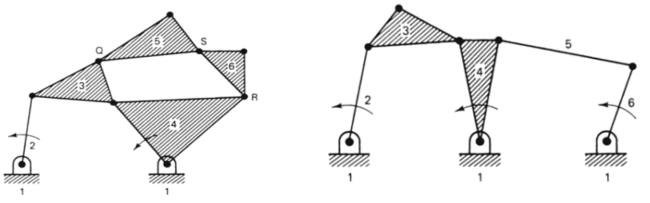

Figure1: Watt I and Watt-II six-bar linkage

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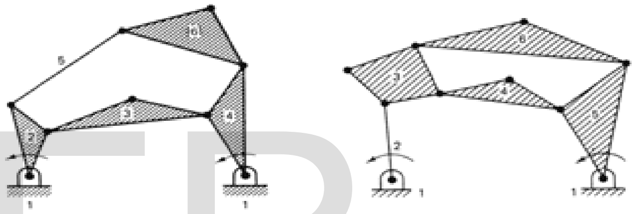

Figure 2: Stephenson I and Stephenson-II six-bar linkage

While the Numerical techniques are commonly combined with various optimization schemes such as genetic algorithms, evolutionary techniques, interior-point method and gauss constrained method. Limitations imposed on size, shape, and Force transmission ability many times require mechanisms that could meet complex design tasks. This paper focuses on the techniques used for optimization of six bar planer mechanism. In first the techniques have been applied on four bar mechanisms and then this work is extended to six bar mechanism.

2 LITERATURE REVIEW

2.1 Synthesis and optimization of four bar mechanisms

Ralph Akhras and Jorge Angeles [1] used unconstrained nonlinear least-square techniques in the optimization of planar linkages for rigid-body guidance. A variable-separation technique, consisting of decoupling the configuration variables from the linkage parameters, was applied here. The number of design parameters, namely eight, was constant, regardless of the number of prescribed configurations of the coupler link. The optimization problem was formulated to lead to an unconstrained over determined system of nonlinear algebraic equations whose least-square approximation is computed by the Newton-Gauss method. Continuation as well as damping techniques was introduced to ensure convergence and enhance its rate. They focused on the production of the linkage having a coupler link attaining a set of prescribed positions and orientations with the least-square error, disregarding further constraints. Wang Zhixing, Yu Hongying, Tang Dewei, Li

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Jiansheng [4] presented the guidance-line rotation method of rigid-body guidance. The method effectively solved the rigid-body guidance synthesis problem for the given more than three coupler positions. Using this method, satisfactory four-bar linkages can be obtained such as crank–rocker mechanism, double-rocker mechanism and double-crank mechanism for the given four, five or more than five rigid- body positions. They also developed a set of software to realize the automatic design and visualization of guidance synthesis for four-bar linkage. H. Schreiber, K. Meer and B.J. Schmitt [5] described a synthesis method where kinematic dimensions can be chosen a priori (to meet side conditions, e.g., a limited area for the locations of the fixed pivots), but do not have to be chosen. A circle point search in combination with homotopy methods was applied and compared two approaches – via the Bezout number and the BKK bound concerning the homotopy methods. . Ahmad A. Smaili et al [10] presented a tabu-gradient search for optimum synthesis of planar mechanisms. Tabu search is used to start a gradient search to drive the solution ever closer to the global minimum. A tabu-gradient algorithm was used to synthesize four-bar mechanisms for path generation tasks. Compared with the corresponding results of the given examples which were generated by other schemes, the tabu-gradient search rendered the most optimal solutions of all. S.K. Acharyya, M. Mandal [20] had been applied three different evolutionary algorithms such as genetic algorithm, particle swarm optimization and differential evolution for synthesis of a four-bar mechanism to minimize the error between desired and obtained coupler curve. The author introduced a new refinement technique for the generation of initial population. Bum Seok Kim and Hong Hee Yoo [34] proposed a unified synthesis method for simultaneously determining the type and dimension of a planar four-bar linkage system to solve rigid body guidance synthesis problems. They employed a spring-connected block model and parameters of the model including block sizes, spring stiffness constants, spring directions, input joint location and coupler point location, as design variables to formulate an optimization problem. Solution of the optimization problem yielded a simultaneous solution for the type and dimension of a planar four-bar linkage system.

2.2 Synthesis and optimization of six bar mechanisms

Charles W. Wampler [2] presented a general method for the analysis of any planar mechanism consisting of rigid links connected by revolute joints. The method combines a complex plane formulation with the Dixon determinant procedure of Nielsen and Roth. They provided numerical solutions and approach facilitated analytical explorations by leading a generalized eigen- value problem of minimal size. They addressed both input/output problems and the derivation of tracing curve equations.

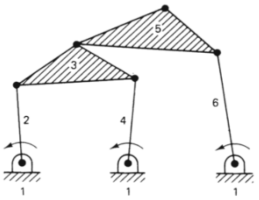

Figure 3: Stephenson-III six-bar linkage

J.A. Cabrera, A. Simon, M. Prado [3] presented the solution methods of optimal synthesis of planar mechanisms. A searching procedure was defined which applied genetic algorithms based on evolutionary techniques and the type of goal function. Four-bar planar mechanism synthesis problems were used to test the method, showing that solutions were accurate and valid for all cases. The possibility of extending the method to other mechanism type was outlined. The main advantages of the method are its simplicity of implementation and its fast convergence to optimal solution.

Shrinivas S. Balli and Satish Chand [6] suggested an analytical method of synthesis of a planar seven-link mechanism with variable topology for motion between two dead-center positions. The proposed non-iterative method is useful to reduce the solution space. S. Mukherjee and S. Sanghi [7] described the design of a flapping mechanism, used to realise the Weis Fogh mechanism of lift generation, used by Micro air vehicle (MAV), a small flight vehicle. A single-drive design using concepts of motion synthesis by kinematic inversion technique, followed by dynamic analysis, was presented. M.A. Laribi et al [8] presented a combined genetic algorithm–fuzzy logic method to solve the problem of path generation in mechanism synthesis. The proposed method was made of a classical genetic algorithm coupled with a fuzzy logic controller (GA–FL) which monitored the variation of the design variables during the first run of the genetic algorithm and modified the initial bounding intervals to restart a second round of the genetic algorithm. They proved their method was more efficient in finding the optimal mechanism. Katsumi Watanabe, Hiroaki Katoh [9] worked on the domains of motion for the driving link of the Stephenson-III mechanism. These domains of motion correspond to either parts of the coupler curve separated by two limit points or connections of two parts separated by limit and turning points, or the coupler curve or its parts separated by two turning points. In their work, these domains of motion were mapped on four number lines, which were discriminated by the sign of the determinant of the Jacobian matrix and the sign of the sine of the relative angle between two links of the external dyad. Consequently, they calculated every relationship between input and output displacements as a continuous functionP.S. Shiakolas, D. Koladiya, J. Kebrle [11] presented a methodology that combines Differential Evolution, an evolutionary optimization scheme, and the Geometric Centroid of Precision Positions technique for mechanism synthesis. Two penalty functions one for constraint violation and one for relative accuracy were employed. The results of the initial application of this methodology were also used in subsequent analysis to considerably improve the desired accuracy level. The developed methodology was applied to the synthesis of six-bar linkages for dwell and dual-dwell mechanisms with prescribed timing and transmission angle constraints. R. Sancibrian, P. Garcı´ et al [12] presented an approximate kinematic synthesis method with general application to path generation, function generation and rigid-body guidance in planar multibody systems. They used exact differentiation for proposed formulation to obtain gradient elements, avoiding the use of numerical differentiation. This method permits the use of many prescribed precision points or poses allowing high accuracy in the definition of the trajectory. Ashok Kumar and Rai Anupam Saxena [13] used curved frame elements within an optimization-based framework for the systematic synthesis of compliant mechanisms (CMs) that can trace nonlinear paths. In the design discretization, the initial slopes at the two nodes of each element were treated as design variables. The proposed synthesis approach used genetic algorithms with both binary and continuous design variables in conjunction with a co-rotational total Lagrangian finite element formulation and a Fourier shape descriptors-based objective function. The objective function was chosen for its ability to provide a robust comparison between the actual path traced by a candidate CM design and the desired path. G. Gatti and D. Mundo [14] proposed a method for the synthesis of cam-integrated six-bar linkages for tasks of exact rigid-body guidance. As a first step, three degrees of freedom six-bar linkage was generated. Two cam-follower mechanisms were then synthesized to reduce the system's mobility and to obtain a single-input cammed-linkage. They also proposed a strategy for the optimization of the synthesis process, based on evolutionary theory. Peter J. Martin, Kevin Russell and Raj S. Sodhi [15] presented an algorithm for selecting planar four-bar motion generators with respect to Grashof conditions, transmission angle conditions and having the minimal perimeter value. This algorithm has been codified into Math CAD for enhanced analysis capabilities and ease of use. Gim Song Soh, J. Michael McCarthy [16] discussed the various options that two RR chains could be attached to constrain the links of the 3R chain, ensured that the end-effector must passed through a set of five specified task positions. This synthesis process yielded designs for the Watt I and Stephenson I, II, and III six-bar linkages except the Watt II because its floating link is not connected to the ground frame by a 3R chain. Qiong Shen, Yahia M. Al-Smadi et al [17] formulated a goal program to generate four-bar mechanism fixed and moving pivot loci that considers prescribed coupler poses, a coupler load and maximum driver static torques. Radovan R. Bulatovic ´, Stevan R. Dordevic´ [18] presented the synthesis of a four-bar linkage in which the coupler point performs approximately rectilinear motion. The motion of the Grashof mechanism crank point located in the prescribed environment of the given point on the observed segment was followed within the prescribed values of allowed deviation. By using the method of variable controlled deviations and by applying differential evolution algorithm (DE), they achieved very high accuracy for motion along a straight line at many given points. Gordon R. Pennock and Ali Israr [19] investigated the kinematics of an adjustable six-bar linkage where the rotation of the input crank was converted into the oscillation of the output link. The analysis used a novel technique in which kinematic coefficients were obtained with respect to an independent variable. They showed how to determine the angle of oscillation of the output link for a specified position of the fixed pivot and investigated the extreme positions of the output link corresponding to the extreme positions of a point on the coupler link. Their study included a study of the geometry of the path traced by a coupler point. Rafael Avilés, Javier Vallejo et al [21] presented an improved approach to the optimum dimensional synthesis of planar linkages based on an elastic strain–energy error function, according to which the optimum link dimensions are those which result in the minimum energy when the linkage is forced to comply with the synthesis data. They mentioned that this method is suitable for any kind of kinematic synthesis for any planar linkage. An overall improvement of convergence had been achieved through a new formulation of the error function and its minimization using two stages in the iterative process. Rafael Avil´es, Javier Vallejo et al [22] presented a method for the generalized rigid-body guidance dimensional synthesis of planar linkages. The error function is based on the elastic energy stored in a finite element model of the linkage, built with rod-type elements, when it is forced to fulfil the synthesis data. The nonlinear deformed equilibrium position was solved in every synthesis position to evaluate the error. The minimization of the synthesis error function was also tackled with two different second-order methods. J.A. Cabrera, A. Ortiz, F. Nadal, J.J. Castillo [23] described an algorithm for synthesis of mechanisms in their work. The algorithm is called MUMSA (Malaga University Mechanism Synthesis Algorithm). They did a comparative study of different strategies for synthesis of four-bar and six-bar mechanisms and obtained the error between the desired and the target coupler curve in a four-bar mechanism and in a six-bar mechanism, showing that the found solutions by the MUMSA algorithm were accurate and valid for all cases. F. Peñuñuri et al [24] demonstrated the optimal dimensional synthesis for planar mechanisms using differential evolution (DE) with four examples. in the first case, the synthesis of a mechanism for hybrid-tasks, considering path generation, function generation, and motion generation, was carried out. The second and third cases pertain to path generation, with and without prescribed timing. They presented the synthesis of an Ackerman mechanism and solved order defect problem by manipulating individuals. Sun Jianwei, Chu Jinkui, et al [25] presented output expressions of linkage mechanism in general mathematical form. The harmonic components of the mathematical expressions were analyzed by using the Fourier series theory. They discovered the relationship between the harmonic component and the harmonic characteristic parameters of coupler rotation-angle operator. The unified derivation process of formula, which can compute the actual dimensions and installation parameters of linkage mechanism, was presented. They set up the unified model for the dimensional synthesis of linkage mechanism from planar, spherical to spatial motion.

Gunesh R. Gogate and Sanjay B. Matekar [26] addressed the problem of optimum synthesis of motion generating four-bar mechanisms using evolutionary methods. They presented formulation of objective functions based on three different error functions, which ensured that the synthesized mechanism is a crank-rocker, and free from branch and order defects. In this, one of the three error functions was an obvious choice, whereas the other two error functions were newly formulated. Differential evolution was used to carry out the optimization for these three objective functions. Radovan R. Bulatović and Stevan R. Đorđević [27] presented the optimal dimensional synthesis of a six-bar linkage with rotational constraints in which a point on the second dyad generated the desired path. The given path was a combination of a rectilinear segment and a circular arc. The authors formed a large population of mechanisms using the method of controlled decrease of allowed deviations and the differential evolution algorithm. Many precision points described the path. The mechanisms were obtained by the method of controlled decrease of allowed deviations with the application of specific adaptation of algorithm DE. Junli Shen, Guoqiang Wang et al [28] established a mathematical model of Z-bar loader mechanism in polar coordinate with four-bar linkage and six-bar Watt linkage synthesis, they investigated the working performance, such as mechanism transmission ratio, carry stability, parallelism, dumping in any position, bucket flat setting, maximum dig depth, extreme transmission angle and so on. In this paper, the writer explored new design methods of joint-position of Z-bar loader linkage between tilt cylinder and loader frame to perfect some of the performance. A new method, comprehensive genetic algorithm, was presented to optimize non-linear equation with multi-constraints, and the results improved the multidisciplinary performance. Prasad Vilas Chanekar, Ashitava Ghosal [29] worked on optimization based method for synthesis of adjustable planar four-bar, crank-rocker mechanisms. For multiple different and desired paths to be traced by a point on the coupler, a two-stage method determined the parameters of the possible driving dyads. In the second stage the remaining mechanism parameters were obtained by using a least-squares based circle-fitting procedure. Shamsul A. Shamsudin et al [30] presented a kinematic procedure to synthesize planar mechanisms capable of approximating a shape change which was defined by a general set of curves called "morphing curves." Their work introduced prismatic joints into the mechanisms to produce the different desired arc lengths. A method was presented to iteratively search along the profiles for locations. They created a chain of rigid bodies connected by revolute and prismatic joints that could be a set of design profiles. Ortiz, J.A. Cabrera, F. Nadal, A. Bonilla [31] proposed a differential evolution with auto-adaptive control parameters which includes a new mutation operator to solve stagnation in local minima and the choice of a control parameters value in a simple way. For a set of 6 representative cases related to dimensional synthesis from bibliography, the performance of this new DE algorithm, called Ingeniería Mecánica Málaga (IMMa) Optimization Algorithm with Self-Adaptive Technique, IOAs-at, has been tested. They also compared the results obtained, with other synthesis techniques. Their new version is auto-tuned during the algorithm execution and not required control input parameters. Gunesh R. Gogate and Sanjay B. Matekar [32] presented unified optimum synthesis of various combination types of Watt-I mechanism, by carrying out unified synthesis the less suited combination types can be identified, leading to their elimination from the synthesis process. This results in a saving of the overall computational time. The presented approach can be implemented with most of the evolutionary optimization methods. In this paper, the Differential Evolution algorithm is chosen as the optimization method. Unified optimization results are presented for two problems. The proposed approach is general and can be used, with suitable modifications, to carry out unified optimum design of alternate mechanical systems which can perform a given task. Gim Song Soh, Fangtian Ying [33] formulated a design methodology for the design of planar six- and eight-bar slider mechanisms for motion generation applications and represented how two RR dyads can be synthesized and attached to planar PRR and PRR–3R chain for the dimensional synthesis of planar six- and eight-bar slider mechanisms, respectively. The results were 15 different types of one degree-of-freedom planar six- and eight-bar linkages with a prismatic joint at its base. The author demonstrated the design process with the design of a multifunctional wheelchair that could transform its structure between a self-propelled wheelchair and a walking guide meant for outpatient rehabilitation purpose. Mark M. Plecnik and J. Michael McCarthy [35] presented a direct solution of the kinematic synthesis equations for Stephenson III six-bar function generators to achieve as many as 11 accuracy points. Their approach is like that used to design Stephenson II function generators, except additional algebraic manipulations reduce the system to a multi homogeneous degree of 55,050,240. They used numerically general multi homogeneous homotopy to obtain 834,441 non-singular solutions. These solutions were used to construct an efficient parameter homotopy for specific tasks consisting of 11 accuracy points. Ping Zhao, Xiangyun Li et al [36] developed a simple algorithm for analyzing a set of given task positions to determine all feasible planar dyads with revolute and prismatic joints. They extended this algorithm to the integrated joint type and dimensional synthesis of Watt I and II and Stephenson I, II, and III six-bar linkages that contain both R- and P-joints. In the process, they developed a new classification for planar six-bar linkages according to whether the end-effector can be constrained by two dyads (type I), one dyad (type II), or no dyad (type III). K. Vikranth Reddy et al [37] presented an analysis of the position kinematics of the complete spatial model of double wishbone (DWB) and the MacPherson strut (MPS) suspension systems. The solution was built upon two key elements: the use of Rodrigue's parameters to develop an algebraic set of equations representing the kinematics of the mechanisms, and the computation of Gröbner basis as a method of solving the resulting set of equations. The depicted configurations of the mechanisms for the real solutions graphically and computed the responses of the suspensions to continuously varying steering and road-profile inputs using a branch-tracking technique. Erkin Gezgina et al [38] presented the design of a hand rehabilitation robot for human four fingers that solely targets hand disabilities by utilizing real grasping motion data of the forefinger into kinematic synthesis of Watt II six-bar linkage. During the design, geometrical synthesis was used for the first loop in order to attain continuous input rotation and the body-guidance synthesis was used for the second loop for the end effector to follow the desired trajectory in desired orientations. S. Sleesongsom and S. Bureerat [39] proposed multi-objective optimization of a rack-and-pinion steering linkage, used in small cars. They assigned a multi-objective optimization problem to simultaneously minimize a steering error and a turning radius. The design variables were linkage dimensions. The problem was solved by the hybrid of multi-objective population-based incremental learning and differential evolution with various constraint handling schemes. The new design strategy leads to effective design of rack- and-pinion steering linkages satisfying both steering error and turning radius criteria. Khalid Nafees, Aas

Mohammad [40] proposed a mechanism with its analytical dimensional synthesis in which two binary links have two offset tracing points. The proposed mechanism transmitted motion between two extreme positions by alternately temporarily fixing a different binary link in two distinct stages. They wrote analytical equations using dyadic and triadic approach of mechanism synthesis for common standard kinematic task of path generation. The proposed method is noniterative and reduces the solution space. Khalid Nafees, Aas Mohammad [41] synthesized a six-bar Stephenson III linkage mechanism of one degree of freedom for 12 precision points. In their approach, loop closure equations were solved simultaneously for 12 displacement positions of coupler tracing point and 12 orientation positions of various links for which the output link oscillates. Displacement vector and coupler link motion were the prescribed parameters. They developed codes in MATLAB to solve these loop closure equations for determination of the dimensional length of each link. Khalid Nafees, Aas Mohammad [42] presented optimal dimensional synthesis of a planar, single degree of freedom, six-bar Stephenson mechanism having rotational constraints. The prescribed parameters of the mechanism were displacement vector and coupler link motion. They determine the optimized dimensions and orientation of various links of the mechanism that is capable to follow the prescribed path. The optimization task for minimum error was carried out with the help of genetic algorithm.

3 CONCLUSION

Several methods have been applied for the synthesis of four bar mechanism and optimization has been achieved using various techniques. The same techniques have been extended to six bar mechanisms of Watt I and II types six-bar linkage as well as Stephenson I, II and III mechanisms all the above methods provides vast opportunities for synthesis and optimization to obtain a required rigid body guidance and a vast area is to be explored.

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